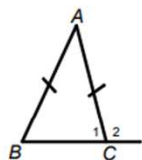
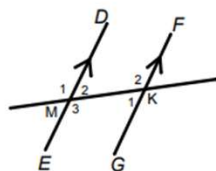


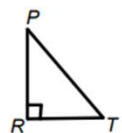
Euclidean Geometry



$\hat{B} = \hat{C}_1$ ($\angle$'s opp. = sides)
 $\hat{A} + \hat{B} + \hat{C}_1 = 180^\circ$ (sum $\angle$'s of Δ)
 $\hat{C}_2 = \hat{A} + \hat{B}$ (ext. $\angle$'s of Δ)



$\hat{K}_2 = \hat{M}_1$ (corres. $\angle$'s DE//GF)
 $\hat{K}_2 = \hat{M}_3$ (alt. $\angle$'s DE//GF)
 $\hat{K}_2 + \hat{M}_2 = 180^\circ$ (co-int. $\angle$'s DE//GF)
 $\hat{M}_1 = \hat{M}_3$ (vert. opp. $\angle$'s)
 $\hat{K}_2 + \hat{K}_1 = 180^\circ$ ($\angle$'s on a str. line)

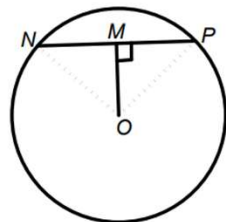


$PT^2 = PR^2 + RT^2$ (Pythag. Th.)

Theorem 1:

(line from centre $\perp$ chord)

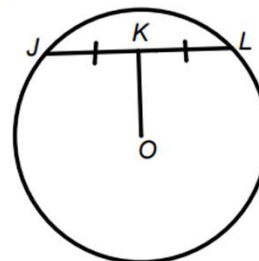
A line drawn from the centre of a circle perpendicular to a chord bisects the chord.



Converse of Theorem 1:

(line from centre mid-pt. chord)

The line segment joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

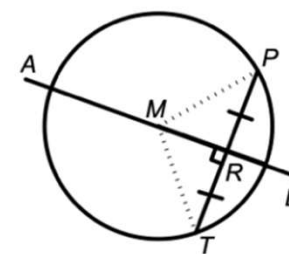


If $JK = KL$, then
 $OK \perp JL$

Converse two of Theorem 1:

(perp bisector of chord)

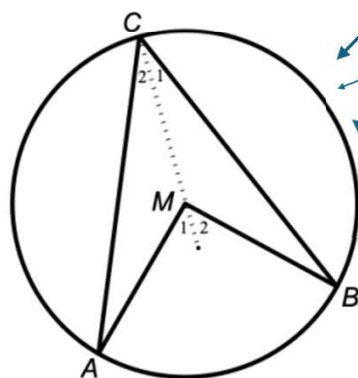
The perpendicular bisector of a chord passes through the centre of the circle.



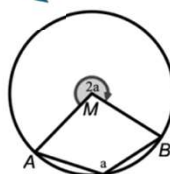
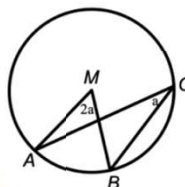
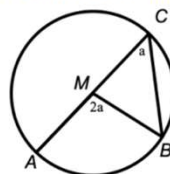
Theorem 2:

($\angle$ at centre = $2 \times \angle$ at circum.)

The angle subtended by an arc at the centre of the circle is twice the angle the arc subtends at any point on the circumference of the circle.



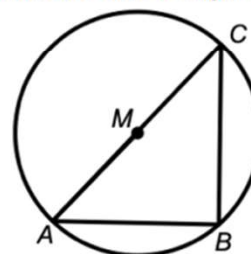
ALTERNATIVE DIAGRAMS:



Theorem 3:

($\angle$ in semi-circle)

The angle subtended by the diameter at the circumference of a circle is a right angle.

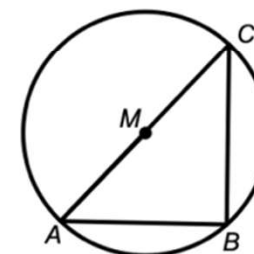


If AMC is the diameter then $\hat{B} = 90^\circ$.

Converse Theorem 3:

(chord subtends 90°)

If a chord subtends an angle of 90° at the circumference of a circle, then that chord is a diameter of the circle.

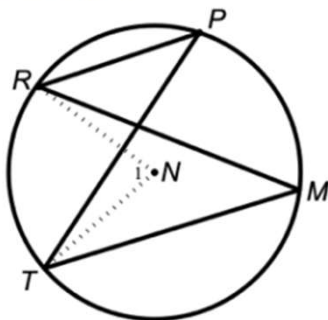


If $\hat{B} = 90^\circ$ then AMC is the diameter.

Theorem 4:

($\angle$ in same seg.)

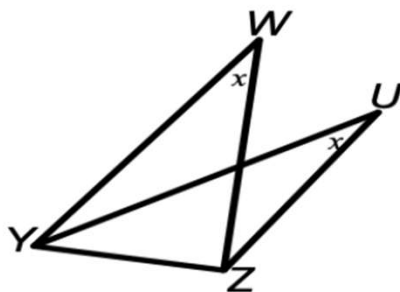
Angles subtended by a chord (or arc) at the circumference of a circle on the same side of the chord are equal.



Converse Theorem 4:

(line subt. = $\angle$'s)

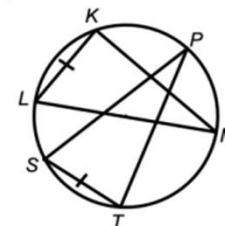
If a line segment joining two points subtends equal angles at two other points on the same side of the line segment, then these four points are concyclic (that is, they lie on the circumference of a circle.)



If $\hat{W} = \hat{U}$, then $WUZY$ is a cyclic quadrilateral.

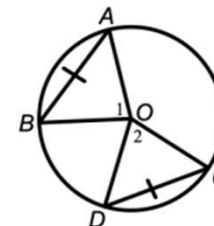
COROLLARIES:

a) Equal chords (or arcs) subtend equal angles at the circumference.



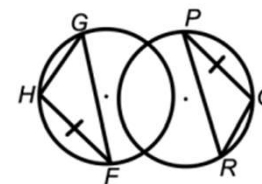
$KL = ST$ then $\hat{P} = \hat{M}$ (= chords, = $\angle$'s)

b) Equal chords subtend equal angles at centre of the circle.



If $AB = CD$ then $\hat{O}_1 = \hat{O}_2$ (= chords, = $\angle$'s)

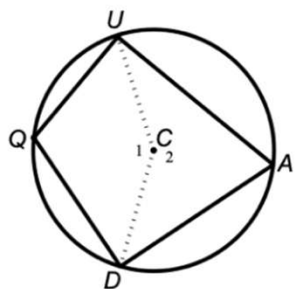
c) Equal chords in equal circles subtend equal angles at their circumference.



If $HF = PQ$ then $\hat{G} = \hat{R}$ (= chords, = $\angle$'s)

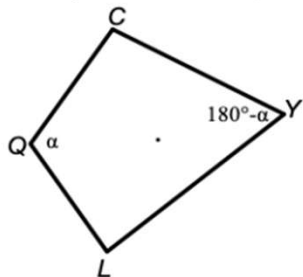
Theorem 5:**(opp. $\angle$'s cyc. quad)**

The opposite angles of a cyclic quadrilateral are supplementary.

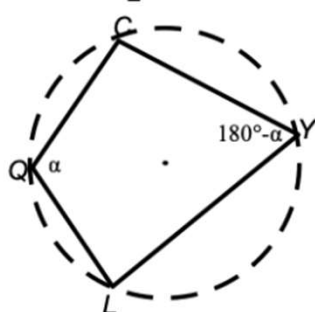
**Converse Theorem 5:****(opp. $\angle$'s quad supp)**

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

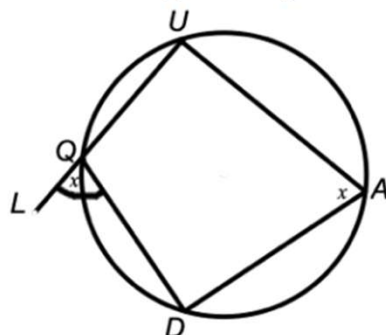
If $\hat{Q} + \hat{Y} = 180^\circ$
or $\hat{C} + \hat{L} = 180^\circ$



Then QCYL
is cyclic

**Theorem 6:****(ext. $\angle$ cyc quad)**

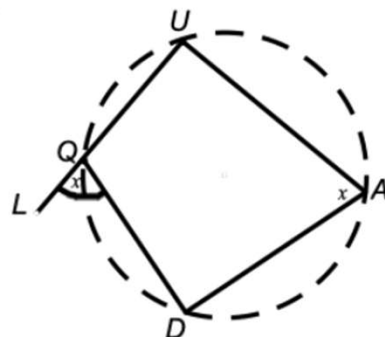
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.



$L\hat{Q}D = \hat{A}$ (ext. $\angle$ cyc quad)

Converse Theorem 6:**(ext. $\angle$ = int. opp. $\angle$)**

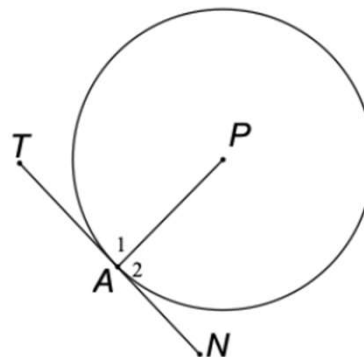
If the exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral is cyclic.



If $L\hat{Q}D = \hat{A}$ then QUAD is cyclic

Theorem 7:**(tan $\perp$ radius)**

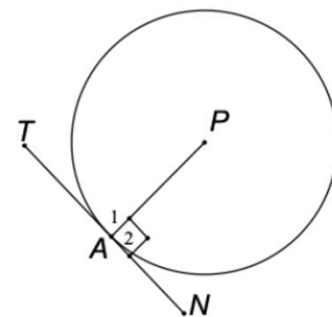
A tangent to a circle is perpendicular to the radius at its point of contact.



If TAN is a tangent to
circle P, then $PA \perp TAN$

Converse Theorem 7:**(line seg $\perp$ radius)**

A line drawn perpendicular to the radius at the point where the radius meets the circumference is a tangent to the circle.

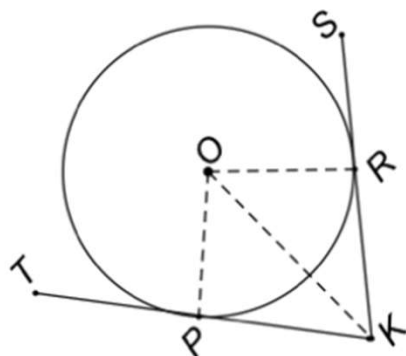


If $PA \perp TAN$, then TAN is a
tangent to circle P.

Theorem 8:

(tan from same pt.)

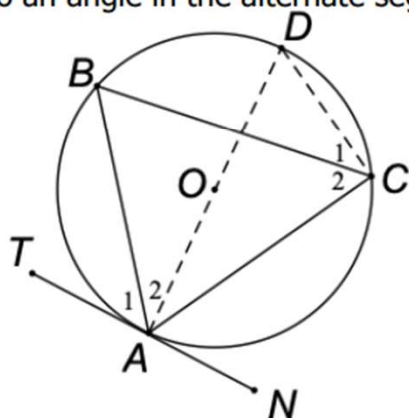
Two tangents drawn to a circle from the same point outside the circle are equal in length.



Theorem 9:

(tan-chord th.)

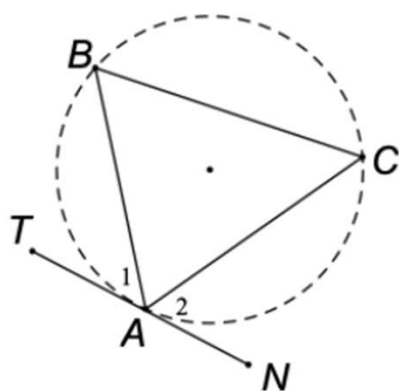
The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.



Converse Theorem 9:

($\angle$ betw. line and chord)

If a line is drawn through the end point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.



If $\hat{A} = \hat{C}$ or $\hat{A}_2 = \hat{B}$,
TAN a tangent

Hints when answering Geometry Questions

- Read the given information and mark on to the diagram if not already done.
- Never assume anything. If not given or marked on diagram is not true unless proved.
- As you prove angles equal or calculate angles mark them on to the diagram and write down statement and reason there and then.
- Make sure that by the end of the question you have used all the given information.
- If asked to prove something, it is true.

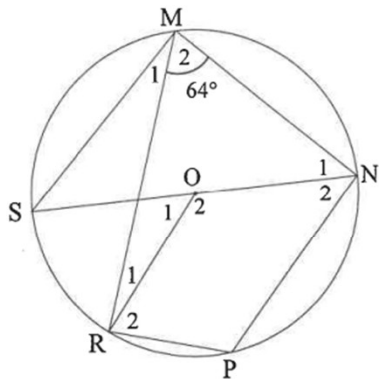
For EXAMPLE if asked to prove ABCD a cyclic quad, then it is; but if you can't then you can use it as one in the next part of the question.

Note for Matric:

All Grade 11 Theorems and their required proofs are also examinable in Grade 12

QUESTION 8

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle. Chord MS and radius OR are drawn. $\hat{M}_2 = 64^\circ$.



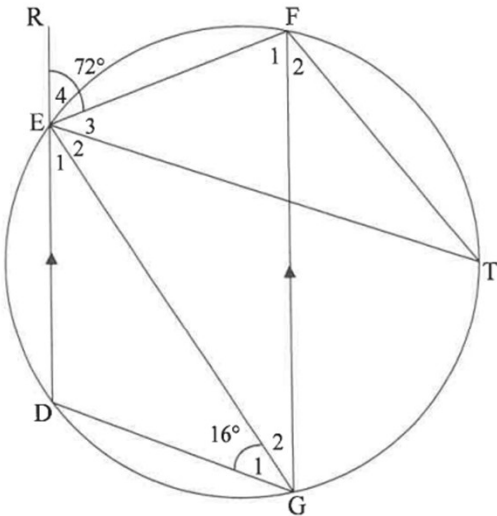
Determine, giving reasons, the size of the following angles:

- 8.1.1 $\hat{P}$
- 8.1.2 $\hat{M}_1$
- 8.1.3 $\hat{O}_1$

(2)
(2)
(2)

QUESTION 9

9.1 In the diagram, DEFG is a cyclic quadrilateral with $DE \parallel GF$. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72^\circ$ and $\hat{G}_1 = 16^\circ$.



Determine, with reasons, the size of the following angles:

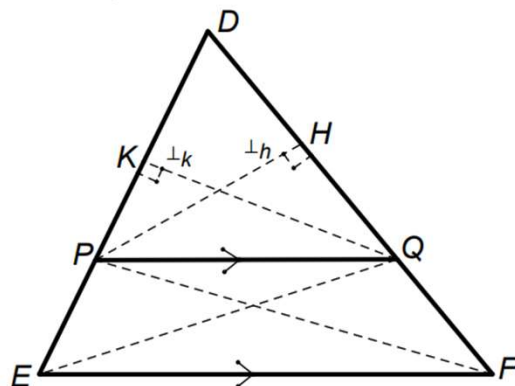
- 9.1.1 $\hat{DGF}$
- 9.1.2 $\hat{T}$
- 9.1.3 $\hat{GEF}$

(2)
(2)
(2)

Theorem 1:

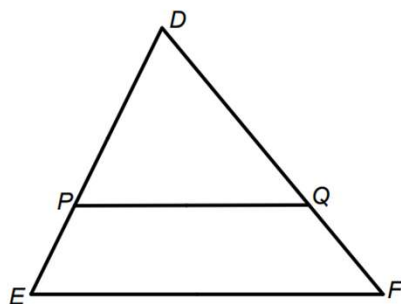
(Proportion Theorem)

A line drawn parallel to one side of a triangle cuts the other two sides in proportion. (Proportion Th. or side 1 $\parallel$ side 2 or line $\parallel$ one side $\triangle$)



Converse of Theorem 1:

If a line divides two sides of a triangle in proportion, then the line is parallel to the third side of the triangle. (line divides 2 sides $\triangle$ in prop. or conv. Prop. Th.)

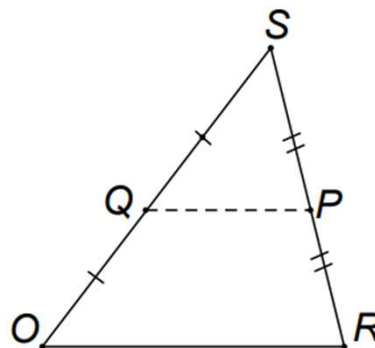


Given $\frac{DP}{PE} = \frac{DQ}{QF}$, therefore $PQ \parallel EF$.

Mid-Point Theorem:

(mid-pt. Th.)

The line segment joining the mid-points of two sides of a triangle, is parallel to the third side and half the length of the third side.

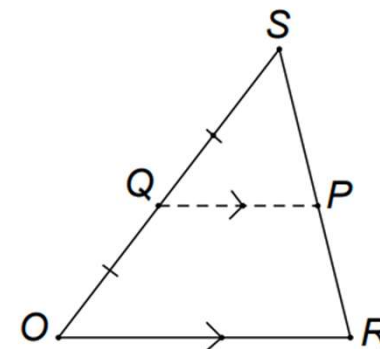


Therefore if $SQ = QO$ and $SP = PR$ then $PQ \parallel OR$ and $QP = \frac{1}{2}OR$ (mid-pt. Th.)

Converse:

(conv. mid-pt. Th.)

The line passing through the mid-point of one side of a triangle and parallel to another side, bisects the third side. The line is also equal to half the length of the side it is parallel to.

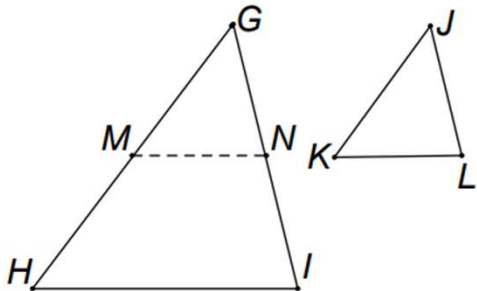


Therefore if $SQ = QO$ and $PQ \parallel OR$ then $SP = PR$ and $QP = \frac{1}{2}OR$ (conv. mid-pt. Th.)

Theorem 2: Similarity Theorem

(AAA)

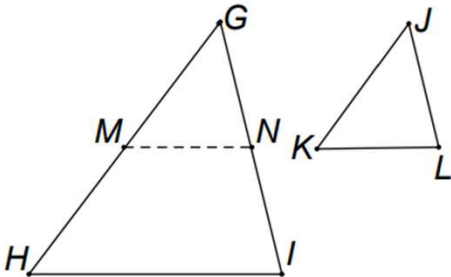
If the corresponding angles in two triangles are equal, then the corresponding sides are in proportion (AAA)



Converse Theorem

(sides in prop.)

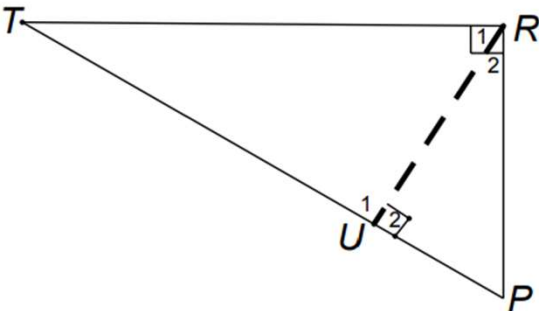
If the corresponding sides of two triangles are in proportion, then the corresponding angles are in equal (sides in prop.)



Theorem 3: Similar Right Angled Triangles

(⊥ from rt ∠ vert. to hyp.)

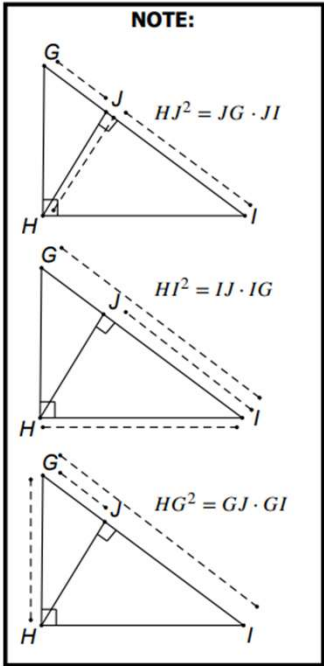
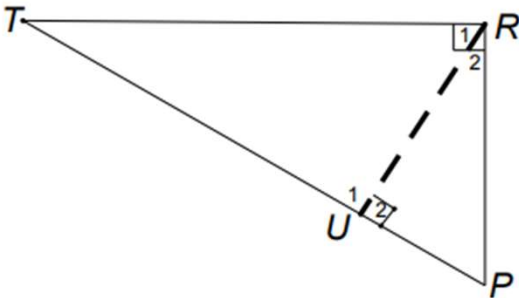
The perpendicular drawn from the vertex of the right angle of a right angled triangle to the hypotenuse divides the triangle into two similar right angled triangles, which are similar to the original triangle. (⊥ from rt ∠ vert. to hyp.)



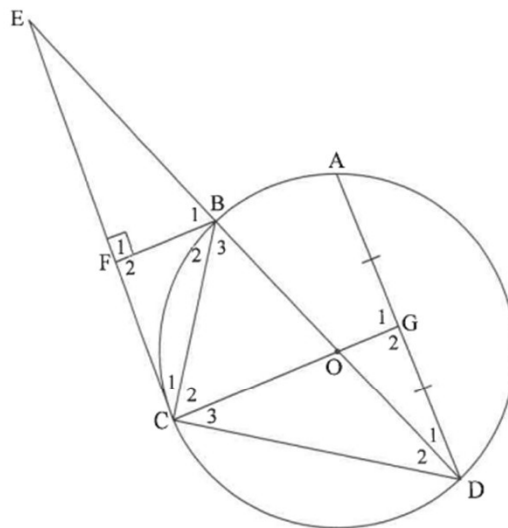
Theorem 4: Theorem of Pythagoras

(Pythag.)

In a right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas on the other two sides. (Pythag.)



- 10.2 In the diagram, O is the centre of a circle passing through A, B, C and D. EC is a tangent to the circle at C. Diameter DB produced meets tangent EC at E. F is a point on EC such that $BF \perp EC$. Radius CO produced bisects AD at G. BC and CD are drawn.



- 10.2.1 Prove, with reasons, that:
- $FB \parallel CG$ (3)
 - $\triangle FCB \parallel \triangle CDB$ (5)
- 10.2.2 Give a reason why $\hat{G}_1 = 90^\circ$. (1)
- 10.2.3 Prove, with reasons, that $CD^2 = CG \cdot DB$. (5)
- 10.2.4 Hence, prove that $DB = CG + FB$. (5)
- [25]